Hydromagnetic Theory of Solar Sectors: Slow Hydromagnetic Waves

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Magnetic sectors on the sun are a feature, when the solar dipole field is subtracted, reminiscent of grapefruit sections in terms of the boundaries described by the magnetic field polarity change. One possible suggestion for the origin of these sectors is that they are hydromagnetic waves controlled by the rotation, toroidal magnetic field, and stratification within the convection zone of the sun. The merits of this suggestion are evaluated with respect to the observations and a specific theoretical model.

I. Introduction

SECTORS are a persistent long-term feature of the solar magnetic field first identified using space probe data for the interplanetary magnetic field. The space probes, which have been confined to a region within $\pm 7°$ of the solar equator but which have sampled distances from 0.7 to 5.0 AU from the sun, observe that the magnetic field is typically pointed towards the sun for several days and then away from the sun for several days, alternating from two to perhaps four times during one solar rotation of 27 days (Fig. 1). The field is somewhat erratic so that it is only on the average that the field within any one period of time is either towards or away from the sun. These intervals of a given polarity are known as "sectors" by their discoverers.[1] Because this feature rotates with the sun, the sectors correspond to intervals of longitude on the sun, near its equator. Sectors have been observed for several years now,[2] and it has been found that although their number and width may change, they are always present. Relating sectors to features on the sun and understanding how those features are distributed spatially and temporally is presently an active area of research.[3-5]

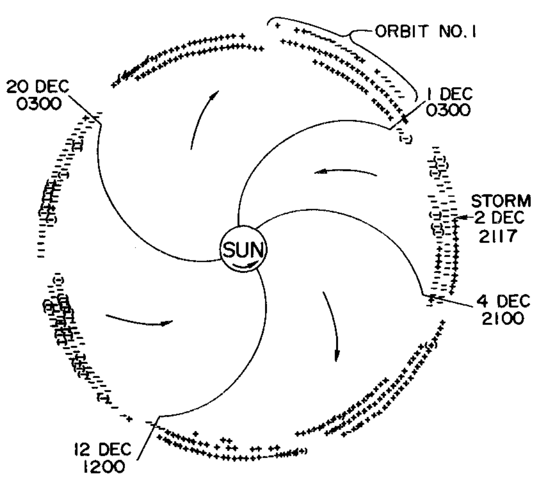

Fig. 1 Plus signs (away from the sun) and minus signs (toward the sun) at the circumference of the figure indicate the direction of the measured interplanetary magnetic field during successive 3-hour intervals. Parentheses around a plus or a minus sign indicate a time during which the field direction has moved beyond the "allowed regions" for a few hours in a smooth and continuous manner. The inner portion of the figure is a schematic representation of a sector structure of the interplanetary magnetic field that is suggested by these observations. The deviations about the average streaming angle that are actually present are not shown. (From Wilcox and Ness.[1])

The question being addressed here is what the internal origin, in the sun, of sectors might be. It has long been accepted that two characteristic features of the sun's magnetic field are a weak axial dipolar field and a strong toroidal field which reverses sign across the equator. Both these fields then reverse every 11 years and are assumed to be a consequence of the combined effects of convection and differential rotation acting through dynamo generation of the sun's over-all magnetic field. The toroidal field is only indirectly inferred through leading sunspot group polarities, and the dipole field has only recently been directly confirmed by making the difficult observation of the field in the polar regions of the sun.

Conversely, sectors are an easily identifiable feature of the interplanetary field and of the mean (averaged over the solar disk) photospheric field of the sun. Their characteristics seem well behaved in time and space and they seem to be a much more stable feature than even the dipole.

Sectors could easily be a consequence of the dynamo processes in the sun.[6] However, it is a distinctly different possibility which will be examined here. This possibility is that sectors may represent large scale hydromagnetic waves in the sun. At the present, neither observation nor theory is sufficiently well developed to cause this mechanism for the production of sectors to be preferred over a dynamo mechanism. However, if sectors could be related to either of these mechanisms, it would be extremely fruitful in terms of producing a valuable observational tool which, through use of a theoretical model, could allow us to learn considerably more about the internal magnetic and angular velocity fields in the sun than we now know.

Previously, an attempt has been made to relate large scale velocity features on the sun to hydromagnetic planetary waves.[7] This effort was highly speculative because of the weak applicability of the theory employed[8] and, more so, because of the difficulty of finding large scale, long-lived features in the velocity field. However, with the development and acceptance of sector observations, new impetus has been given for finding acceptable analytic studies to explain the observations.

Since sectors are seen in the magnetic field at least as well as in the velocity field, the theoretical models should exhibit hydromagnetic waves in the presence of a large toroidal magnetic field and rotation. Additional features to add to a theoretical model might be stratification, differential rotation and specific retention of the effects of spherical geometry. A large class of

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such studies has been developing over the past several years, mainly in reference to dynamics of the core of the earth. Important publications include those of Hide,[9] Malkus,[8] Stewartson,[10] Braginsky,[11] Rickard,[12] and Acheson.[13-14] Specific reference is made to the sun in studies by Gilman[7] and Kato and Nakagawa,[15] the latter of which may reproduce the results of Hide.[9]

II. Slow Hydromagnetic Waves

Observations of sectors suggest that they rotate at an angular velocity differing from some "mean" angular velocity of the photosphere by not more than approximately 5%.[16-17] In surveying the literature just mentioned, perhaps the best candidate then becomes the so-called "slow hydromagnetic waves" detailed in the very complete studies of Acheson.[13,14,18] For these waves, it is extremely important to know how sectors move with respect to the rotation of the sun. According to Acheson's work a slow wave pattern may in general be expected to have an angular wave speed less than the maximum angular velocity of the fluid through which it moves. This does seem to be satisfied by sectors if the maximum angular velocity of the fluid is greater than or equal to that of sunspots. Sectors, however, move more rapidly than matter at the surface of the photosphere. Unlike Alfven waves, slow waves travel at angular velocities not far different from that of the sun, and are more easily generated in large scale lengths. Also, the slow hydromagnetic waves require toroidal magnetic fields not unlike those suspected to exist in the sun if they are to behave like sectors, whereas nonmagnetic and Rossby planetary type waves[12,19] would probably have angular velocities considerably different than sectors and the rotation of the sun and thus be heavily damped.

The simplest derivation of a dispersion relation for slow hydromagnetic waves is for plane waves in a uniform magnetic field with no differential rotation or stratification.[20] In this case, the momentum equation, written in the rotating frame of reference, reduces to

$$(\partial \mathbf{u}/\partial t)+(\mathbf{u}\cdot\nabla)\mathbf{u}+2\mathbf{\Omega}\times\mathbf{u} = -\nabla(p/\rho)+(1/\mu\rho)(\nabla\times\mathbf{B})\times\mathbf{B}+\nu\nabla^2\mathbf{u} \tag{1}$$

in rationalized mks units. μ, ν, ρ, $\mathbf{u}$, and $\mathbf{B}$ are the magnetic permeability, kinematic viscosity, density, and vector velocity and magnetic fields, respectively. $\mathbf{\Omega}$ is the angular velocity of the fluid and Φ is the gravitational potential. The other equations are

$$\nabla\cdot\mathbf{u}=0 \tag{2}$$

$$(\partial\mathbf{B}/\partial t) = \nabla\times(\mathbf{u}\times\mathbf{B})+\eta\nabla^2\mathbf{B} \tag{3}$$

and

$$\nabla\cdot\mathbf{B}=0 \tag{4}$$

$\eta=(\mu\sigma)^{-1}$, where σ is the electrical conductivity. Consider small amplitude plane wave solutions to these equations when the fluid extends indefinitely in all directions with a uniform basic undisturbed magnetic field, $\mathbf{B}_o$. Also take $\nu=\eta=0$. Let $\mathbf{u}_1$, p_1, $\mathbf{B}_1$, be the perturbation wave motion variables. Then, the first-order equations are

$$(\partial\mathbf{u}_1/\partial t)+2\mathbf{\Omega}\times\mathbf{u}_1 = -\nabla(p_{1/\rho})+(\mu\rho)^{-1}(\nabla\times\mathbf{B}_1)\times\mathbf{B}_o \tag{5}$$

$$\nabla\cdot\mathbf{u}_1=0 \tag{6}$$

$$(\partial\mathbf{B}_1/\partial t)=(\mathbf{B}_o\cdot\nabla)\mathbf{u}_1 \tag{7}$$

$$\nabla\cdot\mathbf{B}_1=0 \tag{8}$$

These equations can be combined to give

$$\left\{\frac{\partial^2}{\partial t^2}-(\mathbf{V}\cdot\nabla)^2\right\}\nabla\times\mathbf{u}_1-(2\mathbf{\Omega}\cdot\nabla)\frac{\partial\mathbf{u}_1}{\partial t}=0 \tag{9}$$

where $\mathbf{V}$, the Alfven speed based on $\mathbf{B}_o$, is defined by

$$\mathbf{V}=(\mathbf{B}_o/(\mu\rho)^{1/2})$$

Let

$$\mathbf{u}\sim\exp\left[i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right] \tag{10}$$

This, in Eq. (9), gives

$$\omega^2\pm(2\mathbf{\Omega}\cdot\mathbf{k})(\omega/k)-(\mathbf{v}\cdot\mathbf{k})^2=0 \tag{11}$$

which has the solutions

$$\omega^2=(\mathbf{V}\cdot\mathbf{k})^2+\frac{1}{2}\left[\frac{(2\mathbf{\Omega}\cdot\mathbf{k})^2}{k^2}\pm\left\{\frac{(2\mathbf{\Omega}\cdot\mathbf{k})^2}{k^4}+\frac{4(\mathbf{V}\cdot\mathbf{k})^2(2\mathbf{\Omega}\cdot\mathbf{k})^2}{k^2}\right\}^{1/4}\right] \tag{12}$$

By Eqs. (2) and (4), $\mathbf{u}_1\cdot\mathbf{k}=\mathbf{B}_1\cdot\mathbf{k}=0$, so that the particle displacements and associated magnetic field line motion lie in planes parallel to the wavefronts. Denote by ω_+ and ω_- the solutions for $(+)$ or $(-)$, respectively, as taken in Eq. (12). There are several limits in which Eq. (12) can be usefully examined. However, only one is of immediate interest here. Consider the case when

$$(k^2(\mathbf{V}\cdot\mathbf{k})^2/(2\mathbf{\Omega}\cdot\mathbf{k})^2)\ll 1 \tag{13}$$

which is approximately true in the sun using a wave number, $\mathbf{k}$, defined by that for sectors, Ω the angular velocity of the sun, and an Alfven speed of 85 m/sec. Then, $\omega_+{}^2\gg\omega_-{}^2$, with

$$\omega_+{}^2\approx[(\mathbf{\Omega}\cdot\mathbf{k})^2/k^2] \tag{14}$$

$$\omega_-{}^2\approx[(\mathbf{V}\cdot\mathbf{k})^4k^2/(2\mathbf{\Omega}\cdot\mathbf{k})^2] \tag{15}$$

Equation (14) is just the ordinary inertial wave.[21] However, Eq. (15) is a "hydromagnetic-inertial" wave,[22] or the slow hydromagnetic wave of Acheson. It is characterized by the "magnetostrophic" balance of forces[23] in which the Coriolis, Lorentz and pressure gradient forces are comparable. This is a case thought to often exist in naturally occurring hydromagnetic dynamos.[24,23] These slow waves are highly dispersive, with $\partial\omega/\partial\mathbf{k}$ generally dependent on $\mathbf{k}$, propagate very much more slowly than either pure Alfven waves or inertial waves, and have the curious property of propagating more slowly relative to the rotating fluid as Ω is increased. From Eq. (7) it is found that the ratio of magnetic to kinetic energy of the waves is given by

$$\frac{\frac{1}{2}\mu^{-1}\mathbf{B}_1{}^2}{\frac{1}{2}\rho\mathbf{u}_1{}^2}=\frac{(\mathbf{V}\cdot\mathbf{k})^2}{\omega^2}\gg 1 \text{ for the sun} \tag{16}$$

Acheson goes on to examine the importance of viscous and ohmic dissipation, finding slow waves are much more weakly damped than Alfven or inertial waves when using parameters suitable to the sun. The relation (16) is very convenient for sectors, giving additional reason why the waves might be more easily observed in magnetic, rather than velocity features.

The main result of interest, however, is Eq. (15). As an example, take a magnetic field of 0.3 webers/m^2 and density of 10 kg/m^3—corresponding, roughly, to 0.85 $R_\odot$. For sectors, $k\approx(2\pi n)/R_\odot$ where n is the number of sectors and $R_\odot$ is the radius of the sun (7×10^8m). Ω is about 3×10^{-6} rad/sec. With n equal to 4, this gives

$$\omega_-\approx10^{-7}\ \text{sec}^{-1} \tag{17}$$

or, about $\Omega/300$. This is clearly within the range of possible values for sectors with respect to some mean angular velocity of the convection zone. To raise ω_- to 10% of Ω would require boosting B_o to 15,000 gauss, a not entirely unacceptable value. Placing the origin of, and evaluation for, the waves nearer the top of the convection zone reduces the amplitude of B_o necessary to have ω_- be $\Omega/10$ since the density drops rapidly near the top of this region. Having B_o confined to a very thin region near the surface of the sun would be more in keeping with the kinematic model of Leighton.[25] However, such a model is probably not required to have a wave speed behavior like that of sectors.

A toroidal magnetic field of 0.2 webers/m^2 (2000 gauss) is often suggested as most likely in the sun[25] by inference from sunspot fields. If this were so, the implication would then be that sectors (if they are slow hydromagnetic waves) originate in a relatively thin region ranging only about 5% below the surface of the sun. Such a case assumes the angular velocity in this region, Ω, is represented by the maximum angular velocity of the surface and that sectors travel with a wave speed differing from Ω by 10%.

This example illustrates how the modeling of sectors as slow hydromagnetic waves could be used to infer internal parameters of the sun. To improve the model, further observations of sector behavior and solar rotation are necessary. Additionally, the dispersion relation (15) needs some improvement to allow for a varying $\mathbf{B}_o$, stratification, differential rotation, and geometrical effects for the behavior of the waves in a spherical shell. Some of these limitations have already been at least partially removed.

Hide[9] and Kato and Nakagawa[15] examined the analog to slow waves in a thin spherical shell. Both studies found the now well-known result that in a thin shell the waves all travel more rapidly than the angular velocity of the fluid. This result may imply that the result of the preceding example is misleading, since there is fairly general agreement that sectors do not travel more rapidly than sunspots (and, by inference, the bulk of the convecting matter in the sun). A depth of only 5% of the solar radius, as found above, could (but probably does not) represent a thin shell. Consequently, it seems likely that if sectors are slow hydromagnetic waves, one or more of the following must be happening. a) The toroidal field is larger than suspected. b) Ω or the difference between Ω and the angular velocity of sectors is different than seems from observations of the surface of the convection zone (photosphere). c) The density structure in the convection zone is not well known. The third possibility is fairly unlikely, as modelling stellar interiors seems to be a well developed science. However, the first and second possibilities are quite acceptable.

In attempting to understand the circumstances in which slow hydromagnetic waves may exist, Acheson[14] examined the stability of an infinitely long liquid filled annulus permeated by an azimuthal field which varies with radius. The system can always be rendered stable to axisymmetric disturbances by sufficiently rapid rotation.[26] However, the system may still be unstable to nonaxisymmetric disturbances if the field increases with radius which corresponds exactly to what is suggested for the sun. There, the toroidal field is suspected to maximize somewhere below the surface and to then decrease towards the bottom of the convection zone. The fact that nonaxisymmetric disturbances are most unstable may also help explain why sectors are seen rather than randomly directed waves. A stronger conclusion here, however, must await exchanging a cylinder for a sphere in the theoretical model.

Acheson,[27] in addition to Malkus,[28] has gone on to examine hydromagnetic waves in the presence of shearing, or differential rotation. The general conclusions here are that shearing of the proper sign will make the waves more unstable and that the waves cannot travel more rapidly than the fastest angular velocity of the fluid.

Finally, Acheson[18,20] has attempted to examine the effects of stratification. In general, he finds that the same types of waves

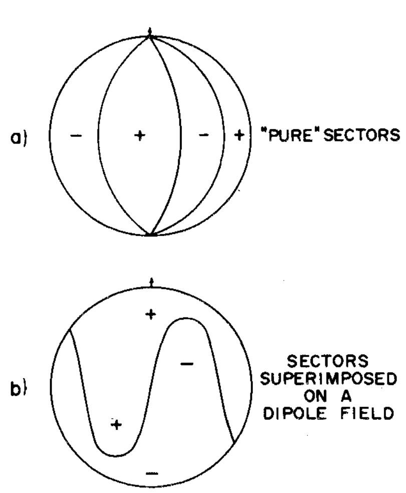

Fig. 2 Two possible configurations for sector boundaries on the sun. a) Schematic representation of magnetic sector polarity change boundaries at the photosphere, on the sun. b) The magnetic polarity change boundaries resulting from superimposing an axial dipole field on the field of Fig. 2a.

exist. However, if the fluid is strongly stably stratified, only high frequency waves (such as Alfven waves) have reasonable growth rates or are not rapidly damped. If, on the other hand, the fluid is convecting or only weakly stable, slow hydromagnetic waves can exist. Clearly, the core of the sun is stable, whereas the convection zone is not. It must then be asked, would the rapid convection of the sun disperse the slow waves to the extent that they would be unobservable? This question is probably answerable in terms of the number of sectors ordinarily observed. Typically, two to four sectors are seen, while the scale of supergranules is such that several thousand of them are visible on the face of the sun. Thus, it would seem that slow hydromagnetic waves of high wave number are dispersed, but that those of low wave number are not, resulting in the observed sectors. This is possible since these large scale sectors would see the very fine pattern of the convection cells as an essentially uniform medium that has a slightly superadiabatic temperature distribution. The convection cells also should turn over with a period substantially shorter than the wave, a condition which is apparently satisfied in the sun where the convective overturn period is on the order of a day, while the sector period is on the order of 10 days. These arguments may not hold if giant cells, comparable to the size of sectors, exist unless the overturn period is substantially longer than 10 days.

III. The Physical Appearance of Sectors

So far, there has been little mention either of how the slow waves might theoretically appear on the sun, or of how sectors appear on the sun. But through several independent lines of evidence, there is now a preliminary model of sector boundary behavior on the sun. Initially, it was shown[17,29] that sectors extend out of the equatorial region of the sun, first by relating sector boundaries observed at the earth to photospheric field reversals and then by showing that the photospheric field reversal occurred simultaneously over a broad range in latitude. This was followed with a more extensive model of the observations by Wilcox and Svalgaard,[27] suggesting the following characteristics for sector boundaries at the sun. 1) The sector structure has a very large extent in latitude on the sun. 2) The boundary, on the sun, is approximately in the north-south direction and shows little or no effect of differential rotation. 3) The polarity of the sector is not influenced by the solar equator.

In support of this view stands the result of Houminer,[31] who found from interplanetary scintillation observations that the boundaries of long-lived streams, in the solar wind, tended to be north-south. High speed long-lived streams in the solar wind tend to be associated with sectors, so the implication is that sector boundaries were also north-south at the time of Houminer observations in September and October of 1971. Houminer also found the scintillation associated with the streams seemed to disappear above 40° in latitude, but this does not necessarily mean the streams (and hence sectors) must be absent above 40°.[32]

Additional complications to the above picture are, however, presented by the results of Rosenberg,[33] Rosenberg and Coleman,[34] Wilcox and Scherrer.[35] They found that the width of sectors is statistically latitude-dependent. This means the boundaries are not exactly north-south, but inclined with respect to meridional planes. Support for this viewpoint, at least during specific intervals within a solar cycle, comes from the observations of Hansen, Sawyer, and Hansen,[3] Howard and Koomen,[5] and Rickett and Coles.[36] Hansen et al.[3] observed the K-corona configuration, Howard and Koomen[5] observed the white light corona, and Rickett and Coles[36] observed interplanetary scintillations, all at different times. Thus, it seems, from a variety of sources, that sector boundaries are often, if not always, inclined.

This has led to a unified model proposed by Svalgaard, Wilcox, and Duvall.[4] Briefly, it is suggested that sectors, by themselves, would have north-south boundaries. However, superposition of sectors onto the underlying dipole field of the sun results in a polarity change boundary reminiscent of the stitching on a baseball. This is illustrated in Fig. 2b. The evolutionary character

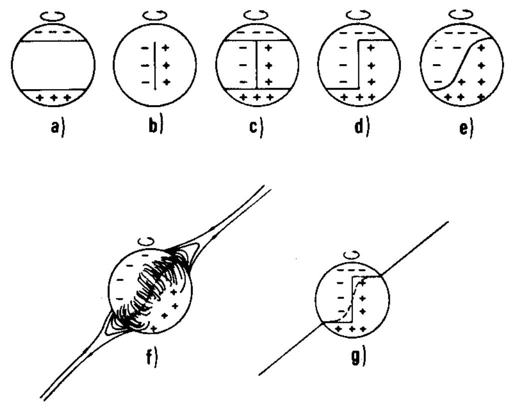

Fig. 3 Stages leading to a model of a solar sector boundary on the solar disk. a) Dipolar polar fields. b) North-south sector boundary, (+, −). c) Superposition of a and b. d) Schematic of boundary between opposite fields. e) More realistic sector boundary. f) Magnetic field line arcade and corresponding helmet streamers in the outer corona over the solar sector boundary. g) Schematic representation of f. (From Svalgaard et al.[4])

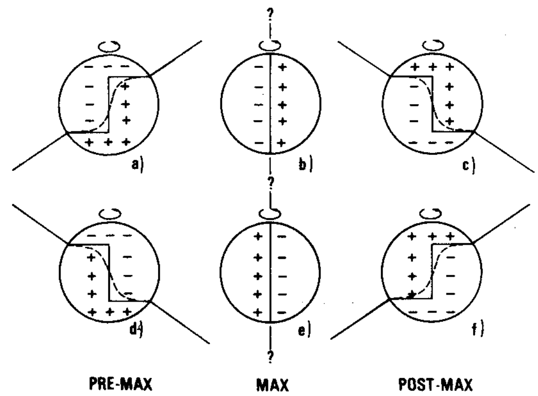

Fig. 4 Sunspot cycle changes in the position of coronal streamers related to solar sector boundaries. The figure is representative for an even-numbered cycle such as the present. (From Svalgaard et al.[4])

of the model is shown in Fig. 3 from Svalgaard et al.[4] Figures 3a and 3b are superimposed to give 3c. Since some polarity change boundaries lose their identity, 3d is the net result. Finally, since nature abhors right angles, 3e is proposed as the actual result. In 3f and 3g, the model is extended to show how it might be physically related to a coronal streamer. Since the dipole field tends to disappear near solar maximum, this interval could result in a "pure" sector. Thus, this leads Svalgaard et al.[4] to suggest the evolutionary succession shown in Fig. 4. Figure 4a–4c represent one hemisphere of the sun, as it proceeds through a solar cycle. Figure 4d–4f represent the opposite hemisphere. The case shown is for two sectors, with additional sectors merely adding geometric complexity. The actual configuration of the polarity boundary depends on the relative strengths of the dipole and sector fields, altering the extent in latitude and longitude and the inclination of the boundary in the region of the equator. Nevertheless, the general features of the diagrams remain identifiable unless the sector field is considerably smaller than the dipole.

Theoretically, the situation is less clear. This is because the calculations of Acheson have not included a case satisfying boundary conditions in a sphere or spherical shell. Some guidance is nevertheless available from related studies. First, Acheson[14] analyzed waves in a fluid filled annulus, with the result that the unstable modes were nonaxisymmetric waves. In addition, Malkus,[8,28] studied hydromagnetic planetary waves in a sphere for a specific field configuration, discovering a large class of azimuthally travelling nonaxisymmetric modes which, in their surface manifestation, could be very reminiscent of the pure sectors of Fig. 2a. Thus, it seems that slow hydromagnetic waves could appear as pure sectors, oriented along the axis of rotation. However, a great deal more work is necessary in this field.

If it can be established that slow hydromagnetic waves do appear as pure sectors, then it would be straightforward to show that superposition on a dipole field results in a configuration like Fig. 3b. It would eventually be necessary to show that the amplitude of the field associated with the sectors is comparable with the dipole field, but this is inherently a nonlinear analysis of the wave behavior. The existence of the solar dipole field is now a fairly well-established fact,[37,38] so that it seems unnecessary to seek a more complex geometrical configuration for the sectors themselves than that implied by Fig. 2a.

IV. Discussion

The whole argument, so far, has been designed to present the slow hydromagnetic waves of Acheson and Hide[20] as an acceptable possibility for the underlying cause of sectors somewhere in the interior of the sun. The reason for seeking the cause is fairly obvious; it supplies a diagnostic tool for examining the interior of the sun. However, there is no intended suggestion that slow hydromagnetic waves are the only possible mechanism for sectors, or that the theoretical models of slow hydromagnetic waves are sufficiently well developed to adequately describe sectors even if it were decided sectors are such waves. In addition, speculation is still limited by the only preliminary knowledge presently available of sector behavior in terms of spatial distribution on the sun and temporal evolution.

With these qualifications, the features of slow hydromagnetic waves which make them attractive for explaining the origin of sectors can be summarized as follows: 1) The wave speed of the waves is comparable to the angular velocity of sectors relative to the observed angular velocity of the photosphere when toroidal magnetic fields like those suspected to exist in the sun are used to calculate the wave speed. 2) The waves would apparently be non-axisymmetric. 3) The energy of the waves is primarily in magnetic field fluctuations.

Improvements needed in the theoretical models for such waves are also fairly obvious. Doing the analysis in a sphere or spherical shell would give considerably more confidence in the applicability of the results. Inclusion of differential rotation and spatially varying magnetic fields would allow a better understanding of the stability of the wave modes. Finally, although it is an extremely difficult calculation, thought should be given to treating a model with a strongly stratified fluid—this last case would be necessary if it became quite likely that sectors are slow hydromagnetic waves.

In conclusion, it appears that it is feasible to propose hydromagnetic models for the origin of solar magnetic sectors. One such model has been suggested here. Other approaches might be through the theory of dynamo generation of magnetic fields (see Ref. 6 for a review). In the final analysis, only a comprehensive comparison of the relative merits of the theory vs observational models can resolve the question of the origin of sectors and allow their use in producing better solar and stellar models.

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